Dynamic Revenue Sharing
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The Revenue Sharing Problem

- Publishers send impressions to AdX with opportunity costs \( c \)
- AdX calls advertisers to bidding for the impressions
- Advertisers submit bids and AdX determines allocations and charges the advertisers
- AdX shares the revenue from the auctions with publishers:
  - Winner pays \( p \)
  - AdX pays \( x \) to the publisher and keeps \( p - x \)

What is the revenue sharing scheme here? Two constraints:

- The payout cannot be less than the cost: \( x \geq c \)
- AdX can take at most \( \alpha \)-fraction of the revenue: \( x \geq (1 - \alpha)p \)

Revenue Sharing Policies

Naïve policy:
- Sets a reserve price \( r = \frac{c}{1 - \alpha} \)
- Sells the impression via second price auction with this reserve
- Pays the publisher \( x = (1 - \alpha)p \)

If the impression is sold, the winner pays at least \( r \) and the payout to the publisher is at least \( c: x = (1 - \alpha)p \geq (1 - \alpha)r = c \)

Let \( b^f, b^h \) denote the highest and second-highest bids.

Let \( \Pi(r, c) \) denote the expected revenue with reserve \( r \) and cost \( c: \Pi(r, c) = E[b \mid b \geq r \cdot (\max(c, b^h) - c)] \)

Single period policy:
- Let \( r^*(c) = \arg \max \Pi(r, c) \)
- Reserve price: \( r^r(c) = \max \left\{ \frac{c}{1 - \alpha} \cdot r^*(c), r^*(0) \right\} \)
- Payout: \( p^*(c) = \max \{c, (1 - \alpha)x\} \)

This policy maximizes the profit of the AdX in all single period policies (i.e., each period is independent with each other).

Note: the AdX may take a less-than-\( \alpha \) fraction of the revenue.

Multiple period policy:
- \( \mu^* \) is the optimal Lagrange multiplier of the dual program.
- Reserve price: \( r^M(c) = r^* \left( \frac{1 - \mu^*}{1 - \mu^* (1 - \alpha)} \cdot c \right) \)
- Payout: \( p^M(c) = (1 - \mu^*)c + \mu^* (1 - \alpha)x \)

This policy maximizes the profit of the AdX in all multiple period policies, where revenue-share constraints are satisfied in expectation.

Heuristic policies:
- Variants of the multiple period policy, satisfying the revenue-share constraints for sure with some loss in profits.

Theoretical Guarantees

- Cumulative-revenue share constraint: AdX takes at most \( \alpha \)-fraction of the total revenue: \( \sum_{t=1}^T p_t(x_t) \geq (1 - \alpha) \sum_t x_t \)
- Prefix-revenue share constraint: AdX takes at most \( \alpha \)-fraction of the prefix revenue at any period: \( \sum_{t=1}^T p_t(x_t) \geq (1 - \alpha) \sum_t x_t \)

Refund policy: pay extra money at the last period to meet the cumulative-revenue share constraint

1. Determine the optimal dual variable \( \mu^* = \arg \min_{\mu \in [0,1]} \phi(\mu) \)
2. for \( t = 1, \ldots, T \) do
3. Set the reserve price \( r_t^r = r^r(\alpha^t(\mu)) \)
4. if item is sold, that is, \( b_t \geq r_t^r \) then
5. Collect the buyers’ payment \( r_t^b = \max(c_t, b_t^r) \)
6. Pay the seller \( p_t^s = (1 - \mu^*)c_t + \mu^* (1 - \alpha^t) \)
7. end if
8. end for
9. Let \( p_T^F = \sum_{t=1}^T \{ b_t \geq r_t^r \} (\mu^t(\alpha^t) - \alpha^t) \) be the floor deficit.
10. Let \( p_T^R = \sum_{t=1}^T \{ b_t \geq r_t^r \} (\mu^t(\alpha^t) - 1 - \alpha^t) \) be the revenue sharing deficit.
11. Pay the seller \( -\min\{p_T^F, p_T^R\} \)

Prefix policy: pay extra money at each period to meet the prefix-revenue share constraint

1. For any revenue sharing scheme \( \langle r^r, p^r \rangle \).
2. Let \( B \leftarrow 0 \) be the bank account of the seller.
3. for \( t = 1, \ldots, T \) do
4. Set the reserve price \( r_t = r_t^r \)
5. if item is sold, that is, \( b_t \geq r_t \) then
6. Collect the buyers’ payment \( x_t = r_t \)
7. Pay the seller \( p_t = \max(c_t, (1 - \alpha)x_t - B, p_t^s(x_t)) \)
8. Update the bank account \( B \leftarrow B + p_t^s(x_t) - (1 - \alpha)x_t \)
9. end if
10. end for

Empirical Study

- Data set: a collection of auction records
- each corresponds to a real time auction for an impression
- a seller (publisher) id + a set of bid records
- each bid record = a buyer id and the bid
- Impressions from 20 large publishers over the period of 2 days (1 day for training and 1 day for testing).
- Train: \( \mu^* \) and \( r^r(\cdot) \) are estimated using the training set.

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<td>Match Rate</td>
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Table 1: Performance of the policies for different \( \alpha \)'s.